# Interest Rates, Commodity Price Changes and Gibson's Paradox

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| **Working Paper Number** | 1983-004B |
| **Revision Date** | January 1983 |
| **Citable Link** | https://doi.org/10.20955/wp.1983.004 |
| **Suggested Citation** | Brown, W.W., Santoni, G., 1983; Interest Rates, Commodity Price Changes and Gibson's Paradox, Federal Reserve Bank of St. Louis Working Paper 1983-004. URL https://doi.org/10.20955/wp.1983.004 |

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# INTEREST RATES, COMMODITY PRICE CHANGES AND GIBSON'S PARADOX

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83-004

*We would like to acknowledge Milton Friedman, the members of the Department of Economics, California State University, Northridge and the members of the Research and Public Information Department of the Federal Reserve Bank of St. Louis for help in improving earlier drafts. We, of course, are solely responsible for any remaining errors. W. W. Brown is Professor of Economics at California State University, Northridge, and G. J. Santoni is a senior economist at the Federal Reserve Bank of St. Louis.

## Abstract

The paper argues that the positive relationship between the level of prices and interest rates noted by Gibson arises, in part, because measured price indexes, which are comprised primarily of the prices of short-lived consumption goods, and nominal interest rates are both driven in the same direction by changes in the real rate of interest.

# INTEREST RATES, COMMODITY PRICE CHANGES, AND GIBSON'S PARADOX

W.W. Brown and G.J. Santoni

## I. Introduction

The positive relationship between the level of measured prices and the nominal interest rate has concerned economic theorists since A. H. Gibson's work in the early part of this century.[1/] Table 1 presents data on the correlation between nominal interest rates and measured price levels for the United States during different time periods. The persistent empirical regularity that Keynes labeled the Gibson Paradox is evident in the results. For all subperiods except one, during which extensive war-time price controls were in place, the correlation between the measured price level and interest rate is positive.

Of course, no empirical observation is paradoxical in itself. Rather, an observation may appear to be a paradox when viewed through a particular theoretical perspective. What makes the above observation troublesome is Fisher's hypothesis that the market interest rate is, roughly, the sum of the *ex ante* real interest rate and the anticipated rate of inflation. In the words of Friedman and Schwartz:

> If price changes were perfectly and instantaneously anticipated, rapid rates of price rise would be associated with high nominal interest rates and low rates of price rise with low nominal interest rates, but there would be no correlation between high prices and high interest rates or between rising prices and rising interest rates.[2/]

Friedman and Schwartz note that despite a recent "narrowing" of the paradox, "it still represents a striking empirical regularity in search of an explanation."[3/]

Fisher, no doubt, engaged in some exaggeration when he claimed "no problem in economics has been more hotly debated than that of the various relations of price levels to interest rates."[4/] However, since the discussion has waxed considerably since Fisher's statement and the present, his hyperbole may have only been slight.[5/]

Our objective in this paper is to contend that a fairly rudimentary factor helps explain the presence of the Gibson phenomenon. Specifically, we argue that, because measured price indexes are primarily indexes of the prices of short-lived present consumption goods, the level of these indexes will be positively correlated with the level of nominal interest rates. This occurs because both the prices of short-lived goods and nominal interest rates are driven in the same direction by changes in the real interest rate. Consequently, the Paradox stems, at least in part, from a failure to carefully distinguish between the general price level of economic theory and various proxies of the price level that are observed.

II. Fisherian vs. Measured Price Levels

Alchian and Klein have noted a significant difficulty in using common price indices as measures of the general level of prices, or "purchasing power of money." They comment that a correct analysis would

> ...base a price index on the Fisherian tradition of a proper definition of intertemporal consumption and <u>leads to the conclusion that a price index used to measure inflation must include asset prices</u> (our emphasis). A correct measure of changes in the nominal money cost of a given utility level is a price index of wealth. If monetary impulses are transmitted to the real sector of the economy by producing transient changes in the relative prices of services flows and assets, (i.e., by producing short-run changes in 'the' real rate of interest), then the commonly used, incomplete, current flow price indices provide biased short-run measures of changes in the 'purchasing power of money.'[6/]

For our purposes, two distinct categories of goods are relevant: Short-lived current consumption goods and long-lived goods which yield an expected flow of future consumption.[7/] Following Alchian and Klein, we take an index of the nominal money cost of a given utility level as a weighted average of the prices of long-lived assets and short-lived present consumption goods and call this the "correct" or "general" price level.

On the other hand, the measured price level, especially in the early periods studied by Gibson, Fisher and Keynes, is based almost exclusively upon the prices of short-lived goods.[8/] In our view, one reason for the Gibson Paradox involves the response of these measured price levels to changes in the rate of interest.

The argument is reasonably straight forward and rests on the well known interest elasticities of the prices of short and long-lived goods. It runs as follows. Assume there is one consumption good, one capital good, and money. Goods are consumed in the present period, $Q^S$, and in future periods. The perpetual annual flow of future consumption is $Q^F$. If the real interest rate is r, the present value (cost) of the consumption stream is:

$$C = Q^S + Q^F/r \quad (1)$$

Equation (1) expresses present value in terms of physical units of the present consumption good. If the nominal price of this good is $P^S$, the nominal value (cost) of the consumption stream is:

$$P^S \cdot C = P^S \cdot Q^S + P^S \cdot Q^F/r \quad (2)$$

The nominal value of the stock of capital goods is $P^S \cdot Q^F / r$ and the community's wealth at the beginning of the period (prior to the consumption of $Q^S$) is $P^S \cdot C$.

If $\lambda^S$ and $\lambda^F$ are the proportions of current period goods and capital goods that are traded through the medium of money during the period, nominal expenditures are:

$$\lambda^S \cdot P^S \cdot Q^S + \lambda^F \cdot P^F \cdot Q^F = P \cdot T = M \cdot V . \qquad (3)$$

where $P^F \cdot Q^F = P^S \cdot Q^F / r$

P = a Fisherian ("correct") index of the general price level

T = the number of transactions between goods and money

M = the stock of money

V = the transaction velocity of money.

Assuming M and T are invariant with respect to the real interest rate, the elasticity of the Fisherian index, P, with respect to r is:

$$\frac{\partial P}{\partial r} \frac{r}{P} = \frac{\partial}{\partial r} \left(\frac{M \cdot V}{T}\right) \frac{r}{P} = \frac{\partial V}{\partial r} \frac{r}{V} \text{ or}$$

$$\eta_r^P = \eta_r^v > 0 . \qquad (4)$$

That is, the elasticity is equal to the interest elasticity of velocity and is positive under the standard assumption that velocity and the real rate are directly related. Consequently, on these grounds, a direct relationship between the level of the real interest rate and the general level of prices presents no theoretical problem. This, of course, has been noted by others (see note 5).

However, the Gibson Paradox arises from the correlation between the interest rate and the <u>measured</u> price level (as distinct from the Fisherian index) and this correlation need not depend on the relationship between velocity (and thereby the general or Fisherian price level) and the interest rate. To demonstrate the point, solve equation (3) for $P^S$ and calculate its interest elasticity <u>under the assumption that the interest elasticity of velocity is zero</u>.

$$P^S = \frac{M \cdot V}{\lambda^S \cdot Q^S} - \frac{\lambda^F \cdot P^F \cdot Q^F}{\lambda^S \cdot Q^S}$$

$$\frac{\partial P^S}{\partial r} \frac{r}{P^S} = - \frac{\partial P^F}{\partial r} \frac{r}{P^F} \frac{\lambda^F \cdot P^F \cdot Q^F}{\lambda^S \cdot P^S \cdot Q^S}$$

$$\eta_r^S = -\eta_r^F \cdot K > 0 \qquad (5)$$

where K is the ratio of the nominal value of expenditures on capital goods to the nominal value of expenditures on present consumption goods.[9] Since the interest elasticity of the present price of capital goods is negative, the relationship between the measured price level and the interest rate is positive even if velocity (and, consequently, the general price level) remains unchanged.

If the standard assumption holds so that velocity responds to changes in r (see note 13), the positive relationship between the measured price level and the interest rate will be more pronounced. Noting that $\eta_r^V = \eta_r^P$ (from equation 4) and that $-\eta_r^F \cdot K = \eta_r^S$ (from equation 5), this is shown in equation (6).

$$\bar{\eta}_r^S = \eta_r^P(1 + K) - \eta_r^F \cdot K > \eta_r^S \tag{6}$$

since $\eta_r^P(1 + K) > 0$

More importantly, the interest elasticity of the measured price level, $\bar{\eta}_r^S$, exceeds the interest elasticity of the Fisherian index ($\eta_r^P$) because the interest elasticity of the Fisherian index receives a relatively heavy weight ($1 + K > 1$) in the above expression and because the present prices of capital goods and the interest rate are inversely related (so that $-\eta_r^F \cdot K > 0$).

Measured price indexes are positively related to the real rate and respond in a more volatile manner to changes in the real rate than a

Fisherian index. These measured indexes rise relative to the Fisherian index when the real rate rises and fall relative to the Fisherian index when the real rate falls.

## III. Some Evidence

### A. The Interest Rate and Narrowly Defined Price Indexes

As a first step, the above argument concerning the greater sensitivity of narrowly defined price indexes to interest rate changes is examined. As the name suggests, the CPI is made up almost exclusively of present consumption goods. The index contains relatively few capital goods, and those that are included receive a relatively low weight.[10/] In contrast, the implicit GNP deflator is a broader index that includes the prices of all currently produced capital goods with their weights being the value of the capital goods in total output.[11/] The deflator weighs capital goods more heavily (about 40 percent more heavily) than the CPI. These two indexes will, of course, be closely related. However, if the above arguments are correct, variation in the interest rate will induce variation in the CPI that is not captured by variation in the deflator. Specifically, an increase in the interest rate will result in an increase in the CPI relative to the GNP deflator and conversely.

To test this proposition, changes in the ratio of the CPI to GNP deflator were regressed on changes in the corporate Aaa bond rate. Specifying the dependent variable in this manner is a crude way of controlling for changes in other factors (money, velocity and output) that have a common influence on both the CPI and deflator. The data are quarterly and span the period I/1965-IV/1984. The results are presented below. The estimate was corrected for first order autocorrelation and t-scores appear in parentheses.[12/]

$$\Delta(\mathrm{CPI/DEF}) = \underset{(1.36)}{.001} + \underset{(2.26)^{*}}{.003}\,\Delta r \qquad (7)$$

$$\bar{R}^2 = .06$$

$$\mathrm{Rho}(1) = \underset{(2.69)}{-.29}$$

The estimated intercept does not differ significantly from zero. As predicted, the coefficient of the interest rate is positive and significant, indicating that changes in the interest rate are directly related to changes in the CPI that are not captured by variation in the more broadly defined deflator.

Equation (7) includes changes in the nominal rate as the explanatory variable, while our explanation turns on changes in the real rate. The nominal rate, of course, varies as both the expected rate of inflation and *ex ante* real rate vary. Both of these sources of variation in the nominal rate are thought to affect velocity and are expected to have a common influence on the CPI and deflator, leaving the ratio unchanged.[13/] If velocity changes were the only channel through which variation in the interest rate changed prices, the expected value of the coefficient of the interest rate in equation (7) would have been zero. However, in addition, variation in the nominal rate that is due to variation in the real rate is expected to be directly related to variation in the CPI relative to the deflator (due to the different interest elasticities of short- and long-lived goods and the different weights each receive in the two indexes) and this is what we find.

## B. Time Series Analysis of Recent Data

The simple correlations presented in Table 1 can be interpreted as evidence supporting the existence of the Gibson Paradox. However, as we have noted above (see especially footnote 5), several authors have raised serious questions regarding the usefulness of simple correlation coefficients in testing for the Gibson Paradox. In particular, Benjamin and Kochin argue that much of the paradox (correlation between the rate of interest and the price level) disappears when the underlying time series are detrended.

Additionally, Friedman and Schwartz, contend that structural changes occurring during the mid-1960s have caused the Gibson Paradox to largely disappear. Briefly, they find that the correlation between the level of measured prices and nominal interest rates has fallen considerably since the mid 1960s. At the same time they find the correlation between the rate of change in prices and nominal rates to have increased dramatically since that time. They contend that it appears "...the markets may have learned their Fisher and so have made Gibson obsolete."[14/] Following Klein,[15/] they argue that what may have caused this change in the relationship between interest rates and measured prices was due to the abandonment of a specie backed monetary standard in favor of a purely fiduciary standard.

In an effort to evaluate these claims, as well as to provide more direct evidence bearing on the contentions we have advanced, we analyze the relationship between the corporate Aaa bond rate, the CPI, and the Dow Jones composite common stock price index using quarterly data from the period I/1965 to

I/1983. Aside from the relative ease of obtaining the data, another factor influenced our choice of the period. This is the observation of Friedman and Schwartz that structural changes have caused the Gibson Paradox to largely disappear since the mid-1960s.

The general approach we follow is to fit a time series model to the respective data series so that the residuals from the "fitted model" are essentially white noise. Such white noise residuals, the so-called (estimated) innovations of the underlying data series, are effectively unrelated to any systematic time dependent variation (e.g., trends, cyclical forces) affecting the series.

We then investigate the correlation between the innovations of the various series. In principle, significant correlations between the innovations of two series would not be due to the series having coincident trends. Rather it would reflect some other factor affecting the respective series.

During the period I/1965 to I/1983, quarterly Aaa bond rates appear to be a first order homogeneous nonstationary process. Examination of the estimated autocorrelation function of the bond rates clearly indicates nonstationarity. However, as would be expected based on the efficient market literature, the bond rate first differences are white noise. That is, the interest rate series appears to be ARIMA (0,1,0). Using the Box-Pierce test procedure based on the estimated autocorrelations of the first differences, we cannot reject the hypothesis that the first differences are white noise at the five percent level.[16/]

The quarterly CPI data appears to be a second order homogeneous nonstationary process. Visual inspection of the autocorrelation functions of the CPI and first differences of the CPI indicate that both arise from nonstationary processes. The autocorrelations for the second differences of the CPI damp out indicating the second differences can be reasonably considered as stationary. However, the second differences of the CPI do not appear to be white noise.[17/] The second and third lags of the estimated autocorrelation function of the second differences suggest that the CPI series can be fruitfully represented as an integrated moving average process. That is, the CPI appears to be ARIMA (0,2,2). The following model was estimated

$$\Delta^2 CPI_t = \underset{(.12)}{.026} + (1 - \underset{(3.04)}{.331B^2} + \underset{(3.03)}{.330B^3})\, E_t \qquad (8)$$

where $\Delta$ is the difference operator and B is a backward shift operator.[18/] The figures in parentheses in equation 8 are calculated t-statistics for the respective estimated coefficients.

Importantly, from our point of view, the residuals of the model in equation 8 appear to be white noise. The Box-Pierce Chi-square statistic based on 24 lags of the estimated autocorrelation function of the residuals, $X^2(3,24)$, is 21.5. As a consequence, the model in equation 8 arguably represents the time dependent variation of the quarterly CPI.

To determine whether there is evidence for the Gibson Paradox during this period, we correlated the first differences of the Aaa bond rate with the residuals from the model in equation 8. Since the first differences and residuals are both white noise, no time dependent variation could be expected to generate spurious correlation between the series. The correlation between the estimated innovations in bond rates and those for the CPI is .33, which is significant at the five percent level. Thus there appears to be evidence that the Gibson Paradox persisted during this period and that structural changes occurring during the mid-1960s did not cause it to vanish. Furthermore, its presence cannot be entirely attributed to spuriousness due to coincident trends in interest rates and the price level.

C. Some Direct Evidence

As indicated above, we have selected the Dow Jones composite index of common stock prices as a proxy for the prices of long-lived goods. Examination of the autocorrelation function of the quarterly data indicates the Dow Jones index is a nonstationary process. However, the autocorrelation function of the first differences suggests that this series is stationary and that the process can be modeled as a first order integrated moving average, ARIMA (0,1,1). The following model was estimated

$$\Delta KPI_t = \underset{(1.03)}{3.96} + (1 + \underset{(2.98)}{.33B})\, E_t \tag{9}$$

where KPI refers to the Dow Jones index and again the calculated t-statistics are in parentheses. The residuals of this model appear to follow a white noise process. The Box-Pierce Chi-square statistic based on 24 lags of the estimated autocorrelation function of the residuals of the model in equation 9, $X^2(2,24)$, is 29.7. The hypothesis that the residuals are white noise cannot be rejected.

The residuals from the fitted ARIMA model for the Dow Jones index were correlated with those for the fitted ARIMA model for the CPI and the first differences for the Aaa bond rate. Our analysis implies that both correlations should be negative.

The correlations are -.26 and -.39 respectively and both are significant at the five percent level.

These results are consistent with our analysis of the Gibson Paradox. They suggest that, in part, the paradox arises because an imprecise proxy for the general level of prices has been employed in studying the relationship between the "price level" and interest rates.

## IV. Conclusions

Our conclusion is that the Gibson Paradox--the positive correlation between the level of market interest rates and the measured price level--is not entirely paradoxical. First, on purely theoretical grounds, if prices are measured correctly, interest rate induced changes in the demand for money (velocity) may produce a positive relationship between the level of interest rates and the level of the correct price index.

Second, as a practical matter, actual measured price levels deviate from the theoretically correct index. In particular, measured price indexes are heavily weighted toward short-lived goods. As a consequence these indexes, as well as nominal interest rates, will move in the same direction as changes in the real rate of interest.

FOOTNOTES

1/ A.H. Gibson, "The Future Course of high Class Investment Values," Banker's Magazine, Volume 115, January 1923, pp. 15-34. As is generally the case in these matters, Gibson was apparently not the first to discover the relationship. Tooke noted the correlation in 1844 (see Robert J. Shiller and Jeremy J. Siegel, "The Gibson Paradox and Historical Movements in Real Interest Rates," Journal of Political Economy, Volume 85, No. 5, October 1977, p. 893) and Wicksell discussed the issue in "The Influence of the Rate of Interest on Prices," Economic Journal, June 1907. Keynes was responsible for labeling the positive relationship "Gibson's Paradox." See John Maynard Keynes, A Treatise on Money (Macmillan, St. Martin's Press, 1971), pp. 177-9.

2/ Milton Friedman and Anna Schwartz, Monetary Trends in the United States and United Kingdom (University of Chicago Press, Chicago 1982), p. 496.

3/ Milton Friedman and Anna Schwartz, Monetary Trends..., p. 546.

4/ Irving Fisher, The Theory of Interest (Augustus M. Kelley, 1965), p. 399.

5/ See, for example, K. Wicksell, "The Influences of the Rate of Interest on Prices," Economic Journal, June 1907; John Maynard Keynes, A Treatise on Money (Macmillan, St. Martin's Press, 1971), pp. 182-4; Robert J. Shiller and Jeremy J. Siegel, "The Gibson Paradox and Historical Movements in Real Interest Rates," Journal of Political Economy, Volume 85, No. 5, October 1977, pp. 891-907; Richard Roll, "Interest Rates on Monetary Assets and Commodity Price Index Changes," Journal of Finance, Volume 27, May 1972, pp. 251-77; Thomas J. Sargent, "Interest Rates and Prices in the Long Run: A Study of the Gibson Paradox," Journal of Money, Credit and Banking, Volume 4, February 1973, pp. 385-449; Gerald Dwyer, Jr., "The Gibson Paradox: A Cross-Country Analysis," Unpublished, Emory University, 1982; Daniel K. Benjamin and Levis A. Kochin, "War, Prices and Interest Rates: A Martial Solution to Gibson's Paradox," A Retrospective on the Classical Gold Standard, 1821-1931, Michael Bordo and Anna Schwartz, ed. (National Bureau of Economic Research, 1984), pp. 587-604; T. Windsor Fields, "Spurious Correlation, Coincident Trends, and the Gibson Paradox: An Empirical Investigation," Unpublished, Miami University, February 1982.

6/ Armen Alchian and Benjamin Klein, "On a Correct Measure of Inflation," Journal of Money, Credit and Banking, Volume 4, February 1973, pp. 173-91.

7/ Of course, there is a spectrum of goods of different durability. For our purposes it is sufficient to consider the extreme cases. Let the present consumption good be one that, once produced, is instantly consumed while the long-lived capital good is one that yields an annual flow of present consumption goods lasting into perpetuity.

8/ For British data the series which have been employed are the Schumpeter-Gilboy (1661-1823), the Gayer, Rostow and Schwartz (1790-1850), and the Sauerbeck-Statist price indexes. These indexes are weighted almost exclusively in the favor of current consumptions goods. A representative example are the relative weights of the Gayer, Rostow and Schwartz index in which current consumption goods receive a weight of roughly, 90 percent. The Schumpeter-Gilboy series (1661-1823) contains an index of Consumers' Goods, Consumers' Goods other than cereals, and Producers' Goods prices. However, the index of Producers' goods prices ends at 1801. Consequently, the index of Consumers' Goods prices was employed in our tests and, presumably, the tests of others which span this period and employ this index. The goods contained in this index are virtually all current consumption goods. In the case of the Sauerbeck-Statist index, the weight of current consumption goods is about 80 percent. None of the above indexes include the prices of finished capital goods. Rather, the durable goods component of the indexes include such items as raw timber, hides, iron, copper, tin and lead. For additional information see Roy W. Jastram, The Golden Constant (John Wiley and Sons, New York, 1977), pp. 190-207; A.D. Gayer, W.W. Rostow and A.J. Schwartz, The Growth and Fluctuation of the British Economy 1790-1850 (Oxford, 1953), Volume 1, pp. 468-70; Elizabeth B. Schumpeter, "English Prices and Public Finance, 1660-1822," Review of Economic Statistics, 1938.

9/ Under the assumption that consumption is the same in each period and that the stream is perpetual so that $P^F \cdot Q^F = P^S \cdot Q^F/r$, $-\eta^F_r = 1$ and $\eta^S_r = K$.

10/ Durable goods have a weight of about 18 percent in the CPI. See Bureau of Labor Statistics, Handbook of Methods, Bulletin 1910.

11/ The sum of the values of durable good production, the production of structures and changes in business inventories averaged about 26.0 percent of GNP during the 1965-84 period.

12/ The regression was checked for second order autocorrelation with negative results (Rho 2 = +.03, t = .26).

13/ The standard assumption regarding the relationship between velocity and the interest rate appears to hold during the data period used here (I/65 - IV/84). The estimated relationship between quarterly changes in the ratio of GNP to M1 and changes in the corporate Aaa bond rate is

$$\underset{}{\Delta(GNP/M1)} = \underset{(4.02)^*}{.03} + \underset{(2.31)^*}{.04}\,\Delta R\ .$$

$\bar{R}^2 = .05$
DW = 2.07

14/ Milton Friedman and Anna Schwartz, Monetary Trends in the United States and United Kingdom (University of Chicago Press, Chicago, 1982), pp. 546.

15/ Benjamin Klein, "Our New Monetary Standard: The Measurement and Effects of Price Uncertainty, 1880-1973," Economic Inquiry, December 1975, pp. 461-83.

16/ Robert S. Pindyck and Daniel L. Rubenfeld, Econometric Methods and Economic Forecasts, Second Edition (McGraw-Hill, 1981), p. 549. The Chi-square statistic, using the Pindyck and Rubenfeld notation, based on 24 lags of the estimated autocorrelation function of the first differences is $X^2(0,24)$ = 26.2.

17/ The Box-Pierce Chi-square test for white noise based on 24 lags of the estimated autocorrelation of the second differences, $X^2(0,24)$, is 50.7.

18/ See Pindyck and Rubenfeld, Econometric Methods..., pp. 528-29.

Table 1
U.S. Data: 1800-1975

| Period and Description | r(i,P) |
|---|---|
| 1) 1800-1836<br>Bimetallism | .559* |
| 2) 1800-1861<br>Bimetallism | .472* |
| 3) 1862-1878<br>Civil War, Suspension | .503* |
| 4) 1879-1896<br>Gold Standard; Resumption | .322 |
| 5) 1836-1896 Excluding 1862-78<br>No Central Bank | .380* |
| 6) 1897-1914<br>Gold Standard; Inflation | .857* |
| 7) 1879-1914<br>Gold Standard; "classic" | .604* |
| 8) 1915-1933<br>Gold Standard; WWI, Depression | .688* |
| 9) 1879-1933<br>Gold Standard; Official | .855* |
| 10) 1934-1946<br>Recovery, WWII | -.679* |
| 11) 1947-1975<br>Recent Experience | .874* |
| 12) 1800-1975<br>Whole Period | .359* |

* significantly different from zero at the 5 percent level